Reading seminar: Birational geometry of foliations

The seminar is usually held on Monday at 10:30 in Aula di rappresentanza.

The main goals of the the seminar are the following:

1. study the basic properties of foliations on surfaces;
2. explain the birational cassification of foliations on surfaces following mainly Brunella [Bru] and McQuillan [McQ];
3. look at more advanced and recent topics, also in higher dimension (this part is yet to be planned).

Oct. 23: Overview and motivation (R. Svaldi). What is a foliation? How do foliations appear in algebraic/birational geometry? Why study foliations? Explain the goals of the seminar: birational classification of foliations on surfaces and its applications.

Oct. 30: Basics on the structure of foliations on surfaces (L. Tasin). Recall the definition of a foliation $\mathcal{F}$ in general (for higher dimensional definitions, see, e.g., [AD, Definition 2.4]). Move to smooth surfaces and following [Bru, 2.1], explain equivalent definitions using vector fields and differential forms.

Explain [Bru, Propositions 2.1, 2.2 and 2.3].

Examples [Bru, 2.3]: fibrations and foliations on $\mathbb{P}^2$.

Nov. 13: Singularities and blow-ups (S. Secci). Following [Bru, Chapter 1 and 2.3(1)], define reduced singularities and the blow-up operation for foliations. In particular explain Theorem 1.1 and compute the blow-up at the origin of the foliations defined by dx, $ydx + xdy$ and $ydx - xdy$ on $\mathbb{C}^2$. Examples: Foliations on the blow-up of $\mathbb{P}^2$ [Bru, 2.3(3)].

Introduce dicritical singularities and explain [Bru, Proposition 1.1].

Nov. 27: Minimal models (P. Chaudhuri). Explain the content of [Bru, Chapter 5], i.e. definition and existence of relatively minimal models and minimal models. See also [McQ] and [PS] for slightly different points of views.

Dic. 4: A rationality criterion by Miyaoka (L. Lombardi). Following [Bru, Chapter 7], explain Miyaoka's theorem on the fact that if $K_{\mathcal{F}} = T^*_{\mathcal{F}}$ is not pseudo-effective, then $\mathcal{F}$ is a foliation by rational curves. The theorem is a crucial step in the proof of the abundance conjecture for threefolds (see [Flips, Chapter 9]).

Dic. 11: Numerical dimension. Explain the content of [Bru, Chapter 8]. Maybe two seminars are needed.

Dic 18 in aula dottorato.: Kodaira dimension. Explain the content of [Bru, Chapter 9]. Maybe two seminars are needed.

??: Index theorems. Explain Baum-Bott formula [Bru, Section 3.1] and Camacho-Sad formula [Bru, Section 3.2]. At least state [Bru, Theorem 3.4].

?? : Boundedness of foliations of general type.

?? : Higher dimensional foliations.

References

[AD] C. Araujo, S. Druel, *On Fano foliations*, Adv. Math. 238 (2013), 70–118.

[Bru] M. Brunella, *Birational Geometry of Foliations*, IMPA Monographs, no.1, Springer.

[McQ] M. McQuillan, *Canonical models of foliations* Pure Appl. Math. Q.4,Special Issue: In honor of Fedor Bogomolov. Part 2(2008), no.3, 877–1012.

[PS] J.V. Pereira, R. Svaldi *Effective algebraic integration in bounded genus* Algebr. Geom.6(2019), no.4, 454–485.

[Flips] *Flips and abundance for algebraic threefolds* (Salt Lake City, UT, 1991) Astérisque(1992), no.211, pp. 1–258.